An analysis of the strength of anisotropic granular assemblies via discrete methods

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Abstract. This paper presents a study on the macroscopic strength characteristics of granular assemblies with three-dimensional complex-shaped particles. Different assemblies are considered, with both isotropic and anisotropic particle geometries. The study is conducted using the Discrete Element Method (DEM), with so-called sphero-polyhedral particles, and simulations of mechanical true triaxial tests for a range of Lode angles and confining pressures. The observed mathematical failure envelopes are investigated in the Haigh-Westergaard stress space, as well as on the deviatoric-mean pressure plane. It is verified that the DEM with non-spherical particles produces results that are qualitatively similar to experimental data and previous numerical results obtained with spherical elements. The simulations reproduce quite well the shear strength of assemblies of granular media, such as higher strength during compression than during extension. In contrast, by introducing anisotropy at the particle level, the shear strength parameters are greatly affected, and an isotropic failure criterion is no longer valid. It is observed that the strength of the anisotropic assembly depends on the direction of loading, as observed for real soils.

Introduction

Recently, the advent of powerful computers has made possible comprehensive simulations of granular assemblies were the dynamics of each grain is individually tracked and its motion numerically integrated. This approach is known as the Discrete Element Method (DEM) which was introduced by Cundall [1]. Considering the significant computational requirements of the DEM, the spherical element has been commonly used to represent grains. Therefore for most of its timespan as a numerical technique to study the micromechanics of granular assemblies, the DEM has been unable to capture the effect of grain shape on the soil macroscopic response.

The authors have introduced a novel and efficient approach to model the collision of grains with general shapes. The method is called the sphero-polyhedra (SP) approach where the shapes are represented by the Minkowsky sum of a polyhedron and a sphere [2]. The method is efficient as simulation with complex polyhedral can take at most 10 times the time of sphere based simulations [2] and it is also capable of model the collision between non convex shapes [3].

In the present study, the SP method is used to explore the failure envelope of different granular assemblies characterized by an anisotropic geometry**.** Further details can be found in [4]**.** The samples are placed inside a virtual True Triaxial Test cells. Then a stress path is programmed with the constraint of constant applied pressure. The sample is sheared along this stress path until it fails and then the failure envelope is observed. The deformation of the failure envelope with higher degrees of grain geometry anisotropy is reported. The contribution of this paper is then a qualitative assessment on the effect of grain shape in this failure envelope.

The Spheropolyhedra (SP) Method

As introduced by the authors [3], the SP method divides a polyhedron representing the particle in a set of vertices, edges and faces, smoothed by a spherical element as shown in Fig. 1. The radius of this spherical element is called the spheroradius. For a general polyhedron only the interactions between vertex and faces and edges and edges are required where contacts are detected and the overlapping distance δ is found. This multi-contact approach is what makes the method a versatile option to deal with complex shapes, including non-convex ones [3].

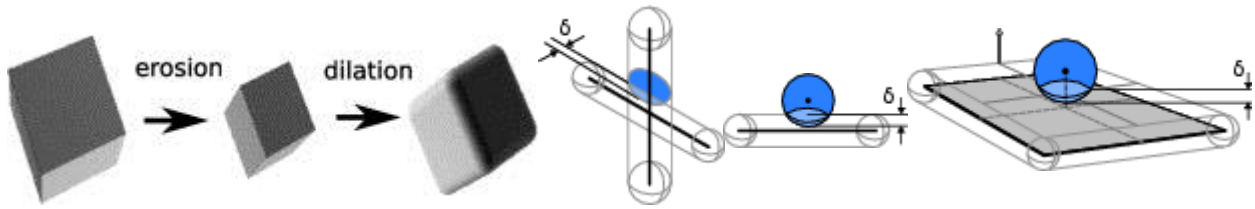

Figure 1. In the left a cube is shrunk and then dilated by a sphere of radius R producing and sphero cube of similar dimensions and rounded corners and edges. Each of this rounded vertices, edges and faces interact with the corresponding feature from the second particle. The possible interaction: edge-edge, vertex-edge and vertex-face are shown on the right. Please refer to [5] for further details.

After finding the contact point for each element, the elastic repulsive force is proportional to the overlapping distance δ:

$$\overrightarrow{F_n} = K_n \delta, \tag{1}$$

where the constant K_n is the normal stiffness. This is the basic contact detection method used for the spheropolyhedra. Once the contact is detected, frictional and dissipative forces are added to model inelastic collisions. Frictional forces are calculated from a tangential displacement δ_t which starts accumulating from the time t_c where the contact started,

$$\delta_t = \int_{t_c}^{t} v_t dt\,, \tag{2}$$

with v_t the tangential velocity of the contact at time t. The value for the frictional force is:

$$F_{friction} = \min\{K_t \delta_t, \mu F_n\}, \tag{3}$$

which depends on a tangential stiffness K_t and the friction coefficient μ. This is the standard Cundall Strack model for static friction[1]. Lastly, viscous forces are added depending on a normal viscosity constant G_n, the effective mass of the particle pair m_e and the normal relative velocity of the particles v_n,

$$F_{viscosity} = G_n m_e v_n, \tag{4}$$

All these forces are added together, and then Newton's second law for translation and Euler's equations for rotation are numerically solved as explained in[5]. Table 1 shows the value for the parameters used in this study

Table 1. Parameter values used in the simulation

Parameters	Value	Unit
Normal stiffness, K_n	100	kN/m
Tangential stiffness, K_t	50	kN/m
Normal viscous coefficient, G_n	0.8	s^{-1}
Friction Coefficient, μ	0.4	1

The Voronoi particle generation

In a previous study [6] the authors introduced the Voronoi spheropolyhedra. The method used the Voronoi tessellation to generate the random geometry of the particles. In this study the original Voronoi tessellation is deformed along the vertical axis to produce particles that appear flat. The general geometry is characterized by the aspect ratio relating the horizontal dimensions with the vertical one. Fig. 2 shows some examples of the Voronoi SP used in this study. The aspect ratio, defined as the average ratio between the vertical and horizontal dimensions of the particles, is denoted herein as Λ.

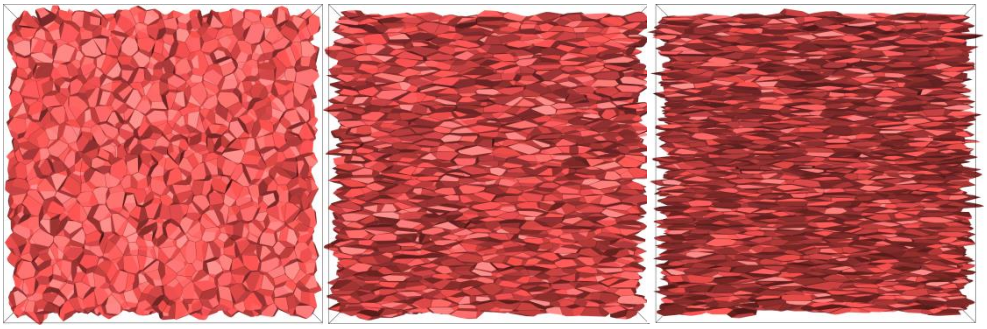

Figure 2 Examples of ensembles with different aspect ratios Λ. From left to right Λ=1,3,5.

The True Triaxial Test (TTT)

The True Triaxial Test (TTT) is an alternative to the typical cylindrical triaxial test. In the TTT the experimentalist can control the stresses and strains in the 3 directions independently. The stress path chosen for this study is the p-constant path where the pressure is kept constant. The stress state could be defined in terms of three quantities: the deviatoric stress invariant q, the pressure p and the Lode angle θ.

The stress p-constant stress path is explained in Fig. 3. Initially the sample is isotropically loaded. Once equilibrium is reached, the sample is sheared keeping the pressure constant and varying the Lode angle.

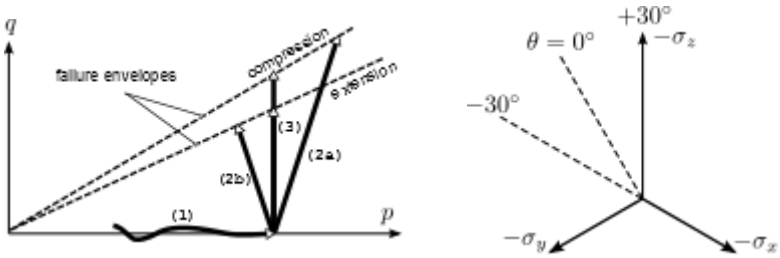

Figure 3. In the p-constant stress path the granular sample is isotropically loaded and then sheared ensuring that the pressure remains constant, path (b) in left figure. The figure on the right illustrates the Lode angle in the octahedral plane.

Simulation Results

Initially, the isotropic sample with Λ=1 is subjected to the TTT plan. The results for several Lode Angles are shown in Fig. 4 in both the q-p and the octahedral planes. The results are compared with three well known functional forms for the failure envelope: the Mohr/Coulomb, the Lade/Duncan and the Matsuoka/Nakai [7]. As can be observed, the best fit for the behavior of the Voronoi SP soils is the Lade/Duncan failure criterion.

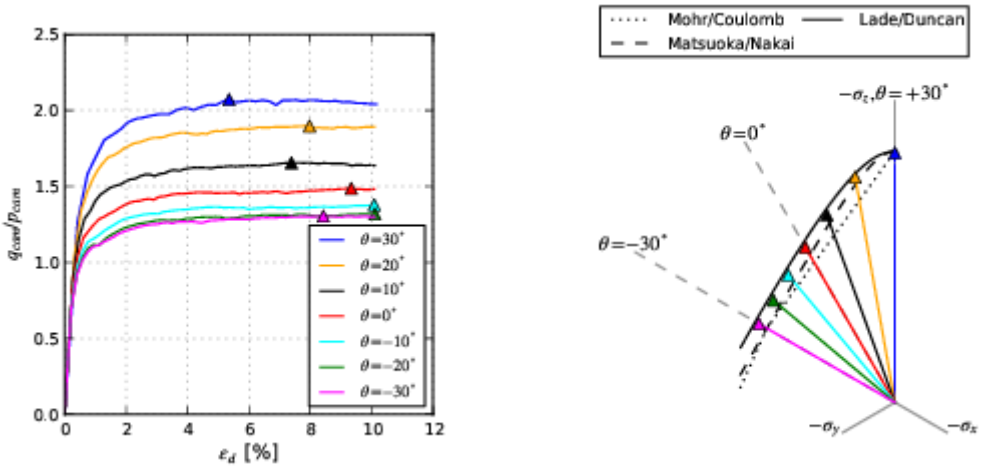

Figure 4. Left, stress ratio as a function of the deviatoric strain showing the failure state. Right, the failure envelope in the octahedral plane compared with three widely used fitting models.

The next stage is to conduct the same TTT plan to the anisotropic samples. Fig.5 shows the results for different values of the aspect ratio. The failure envelope is noticeable deformed with the increasing aspect ratio. It can be observed that the anisotropic failure envelope presents symmetry between the *x* and *y* directions but it is deformed along the *z* direction. For compression along the *z* axis (θ = 30°) there is an increase in the shear strength while for extension (θ = -150°) a slight decrease is detected. This difference between compressions along the horizontal axes is even more pronounced as the aspect ratio Λ is increased.

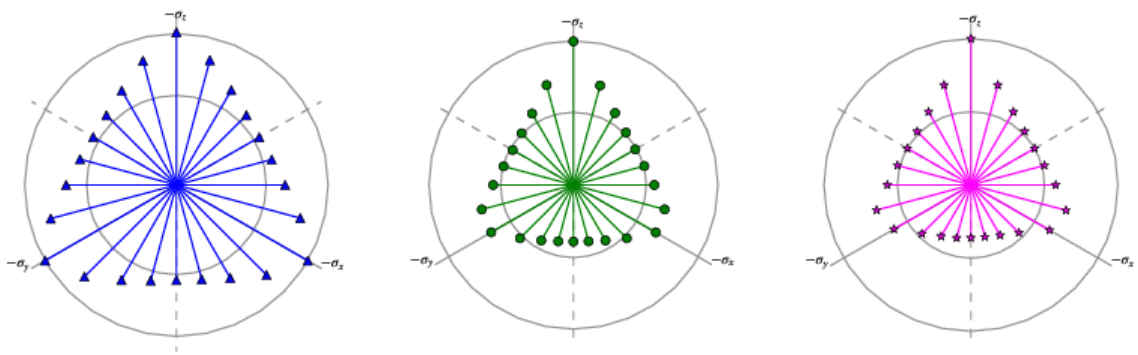

Figure 5. Three failure envelopes for aspect ratios of 1 (isotropic) 2 and 4. The concentric circles are drawn for comparison and visual aid.

The obtained peak stress ratios (M=q/p at the peak) point for four Lode angles are shown in Fig. 6. For the compression along the *z* axis (θ = 30°) M grows with the increase of the anisotropic ratio Λ until it reaches an upper limit M = 3.0 for Λ=4. This corresponds to a friction angle ϕ of 90°. Therefore, with this stress configuration a maximum shear strength parameter is found, unless the particles are allowed to break which is ignored in the present model. In contrast, for extension along the *z* axis (θ = -150°) the shear strength is reduced. Therefore the anisotropic specimen is susceptible to failure when σ_z is reduced with confined conditions. For compression along *y* (θ = -

90°) the shear strength is gradually reduced while for y extension (θ = 90°) it is reduced. Fig 6 shows the comparison between the isotropic and anisotropic failure envelopes showing how these two different soils can be comparatively more or less stable depending on the direction of the applied stress.

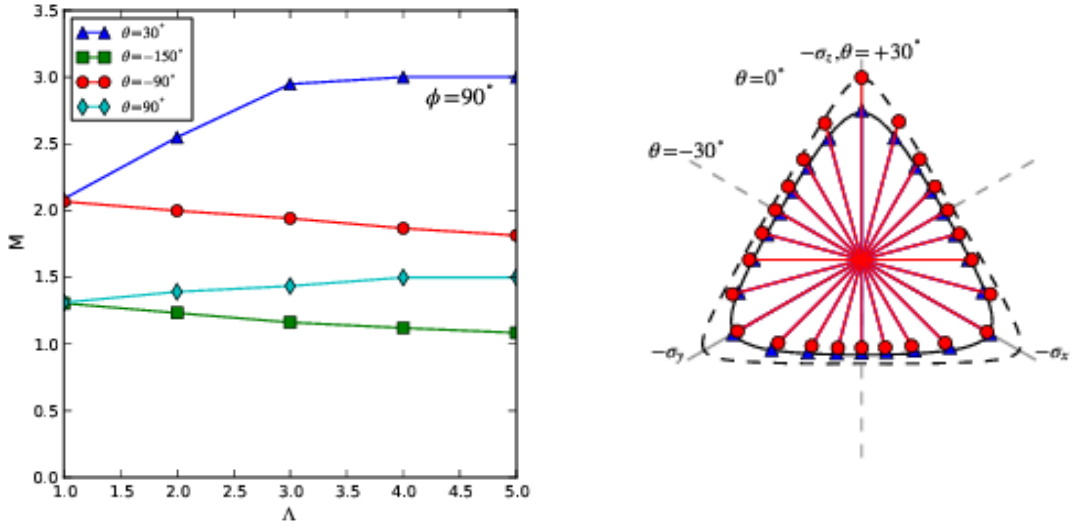

Figure 6 Left, stress ratio M at the peak versus the aspect ratio Λ. Right, a comparison of the failure envelope for the isotropic sample (Λ=1 blue triangles) and an anisotropic sample (Λ=2 red circles)

Final discussion and conclusions

The Voronoi sphero-polyhedra scheme presented herein can be used to study the effect of particle shape with a more sophisticated approach than the rolling resistance models previously used in DEM. The results of simulations are presented for Voronoi sphero-polyhedra. In this paper, both isotropic and anisotropic random Voronoi specimens are considered. It is found that the isotropic Voronoi sphero-polyhedra failure envelope is well-fitted by the Lade Duncan model. It has been found that the failure envelope is strongly affected by particle anisotropy. By inducing anisotropy at the particles geometry, a deformed failure envelope with cross-anisotropy has been reproduced. It is observed that a strong inherent particle anisotropy can theoretically produce packings that have a maximum strength (ϕ = 90°) when the loading is parallel to the direction in which the geometric anisotropy is induced. In this case the largely elongated particles have common contact planes perpendicular to this axis and a maximum stress applied along this direction will not make this contacts to fail, producing this maximum strength. Further studies, with different specimen preparation methods are being carried out. A deposition simulation under gravity should be carried out prior to the triaxial test simulation, to observe if after deposition the shear strength parameters presented in this paper are still valid. Other effects such as tapping and shaking should be introduced as well. If after these simulations the particles' principal planes are still aligned, it is expected that the measured shear strength parameters are the same as the ones reported herein.

References

1. Cundall, P. and O. Strack, *A discrete numerical model for granular assemblies.* Geotechnique, 1979. **29**(1): p. 47-65.
2. Galindo-Torres, S., J. Muñoz, and F. Alonso-Marroquín, *Minkowski-Voronoi diagrams as a method to generate random packings of spheropolygons for the simulation of soils.* Physical Review E, 2010. **82**(5): p. 056713.
3. Galindo-Torres, S., et al., *Molecular dynamics simulation of complex particles in three dimensions and the study of friction due to nonconvexity.* Physical Review E, 2009. **79**(6): p. 60301.
4. Galindo-Torres, S., et al., *Strength of non-spherical particles with anisotropic geometries under triaxial and shearing loading configurations.* Granular Matter, 2013: p. 1-12.

5. Galindo-Torres, S.A., et al., *Breaking processes in three-dimensional bonded granular materials with general shapes.* Computer Physics Communications, 2012. **183**(2): p. 266-277.
6. Galindo-Torres, S.A. and D.M. Pedroso, *Molecular dynamics simulations of complex-shaped particles using Voronoi-based spheropolyhedra.* Physical Review E, 2010. **81**(6): p. 061303.
7. Mortara, G., *A hierarchical single yield surface for frictional materials.* Computers and Geotechnics, 2009. **36**(6): p. 960-967.